Birational coordinates on nilpotent coadjoint orbits of $\mathrm{SO}(N, \mathbb{C})$

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Symplectic spaces are very interesting, they have many applications in different fields of the modern mathematics. One of the sources of a symplectic structure is the Lie-algebraic construction called *a canonical Lie-Poisson-Kirillov-Kostant structure*. It is defined on any coadjoint orbit of any Lie group. The coordinatization of the orbits is an important problem. We consider a complicated case of the orbit of the orthogonal group, the orbit consisting of nilpotent matrices.

In contrast with the nilpotent orbits the construction of the orbits generated by the diagonalizable matrices is simple. The Darboux coordinates on them can be constructed using *projection and contraction* on the isotropic eigenspaces and the co-isotropic co-eigenspaces. The (co)eigenspaces of non-zero eigenvalues have necessary properties.

The kernel of the nilponent matrix is not isotropic but it has isotropic subspaces. The procedure of projections and contractions gives a set of functions connected by non-trivial equations in the nilpotent case. I will demonstrate how to organize the process of projections and contractions in such a way that the corresponding equations will be solved.

Let us define integer numbers δ_k. The number δ_k is the number of the Jordan blocks the sizes of which are greater than $k \times k$, and the sizes of these blocks have the parity opposite to the parity of k. We put $\delta_k = \delta_{-k}$, and consider the integer variables $n, k \in 1-M, 2-M, \ldots, M-1$, where $A^M = 0 \neq A^{M-1}$. Consider any matrix A from the orbit as a block-matrix, formed by the blocks $B_{nk},\ n, k \in 0, \pm 1, \ldots, \pm(M-1)$. The size B_{nk} is $\delta_n \times \delta_k$. Let the indexes $n, k \in 0, \pm i \ldots$ numerate the blocks B_{nk}.

The main observation is the following. If each flight of the process has the smallest size, we get the strictly upper-triangular block-matrix with *the zero blocks along the diagonal* i.e. the blocks having common sides with the diagonal blocks are zero.

The iteration process of the factorization and the contraction produce a pair of the block-triangular matrices. One of them is lower-triangular, it is fixing the flag related with the matrix on the orbit. The other one is upper-triangular block-matrix with the zero blocks along the diagonal. Non-trivial matrix elements of these matrices form the birational coordinates on the orbit.

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