On M.N. Lagutinski method for integration of ordinary differential equations

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Researches of M. N. Lagutinski on the theory of integration of the differential equations were interrupted with his tragic death in 1915; here they are considered from viewpoint of modern computer algebra.

For a ring A with differentiation D and basis

$$B = \{\phi_1, \phi_2, \dots\}$$

the sequence of determinants is entered, they are called further as Lagutinski determinants. The notion of rational integral is entered without assumption about integrity of the ring. If one of Lagutinski determinants is equal to zero, the rational integral exists, moreover, we can always calculate this integral. The converse is proved at additional assumptions concerning the ring R.

The differentiation D will be called as contracting differentiation if there is a basis in which

$$D\phi_i = c_i\phi_i + o(\phi_i), \quad c_i \in c(R).$$

For the differentiation it is possible to write out simple necessary criterion of existence of integrals: among indexes c_i there are equal.

Possibilities of Lagutinski method are illustrated with simple examples in a ring $Q[x, y]$ and $Q[x, y, z]$, including the question about integrals of Brio and Bouquet differentiation. M. Singer's theorem opens a way to application of this method for integration of the differential equations in quadratures.

References

1. *Lagutinski M.N.* Prilozhenie poljarnyh operacij k integrirovaniju obyknovennyh differencial'nyh uravnenij v konechnom vide [The application of polar operations to integration of the ordinary differential equations

in finite terms]. // Soobshh. Har'kov. matem. obshh. 2 ser. 1911. T. 12. P. 111-243. (in Russian)

2. *Lagutinski M.N.* O nekotoryh polinomah i svjazi ih s algebraicheskim integrirovaniem obyknovennyh differencial'nyh algebraicheskih uravnenij [On some polynoms and their aplication for algebraic integration of ordinary differential algebraic equations]. // Soobshh. Har'kov. matem. obshh. 2 ser. 1912. T. 13. P. 200-224. (in Russian)

3. *Dobrovol'skij V.A., Strelcyn J.-M., Lokot' N.V.* Mihail Nikolaevich Lagutinskij (1871 – 1915). // Istoriko-matematicheskie issledovanija. 2001. T. 6 (41). P. 111-127. (in Russian)