Growth and oscillations of normalized dimensions of standard and strict Young diagrams

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We present the results of a computer investigation of asymptotics for maximum dimensions of linear and projective representations of the symmetric group. This problem reduces to the investigation of standard and strict Young diagrams of maximum dimensions. We constructed some sequences for both standard and strict Young diagrams with extremely large dimensions. The conjecture that the limit of normalized dimensions exists was proposed by A. M. Vershik 30 years ago [1] and has not been proved yet.

We studied the growth and oscillations of the normalized dimension function in sequences of Young diagrams. Our approach is based on analyzing finite differences of their normalized dimensions. This analysis also allows us to give much more precise estimation of the limit constants.

We are particularly interested in studying the sequence of Young diagrams of maximum dimensions. Unfortunately, it is extremely hard to obtain such a sequence. However, it is possible to construct the so-called greedy sequence. It is constructed by choosing on each step the branch in the Young graph which leads to the diagram of maximum possible dimension. We are studying greedy sequences and expect that the asymptotics of their dimensions is same as the asymptotics of dimensions of maximum diagrams for both standard and strict diagram cases.

References

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