

# On the computational complexity of the three-body initial value problem

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**Abstract.** A classical example of a chaotic dynamic system is considered as a computational problem, stated in terms of computable real numbers. A hypothetical Turing machine, using an infinite tape containing initial values as an input, and a number  $t$  as a time interval, can calculate the state of the system at time  $t$  as computable real numbers. Known algorithms of numerical integration require at least  $O(t)$  time to complete, even disregarding the precision loss. This work deals with the question: can there exist such a machine that solves the initial value problem for three-body system in polynomial in  $\log(t)$  time? Basing on the works of Sitnikov and Alexeyev, we can deduce that a particular case of the three-body problem is so combinatorially rich that no Turing machine can find the solution in a polynomial time.

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