

Relations between discrete and smooth Morse theories

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Abstract. We work with discrete Morse theory, that is a discrete analogue of the classical Morse theory. It was developed by R. Forman [1]. This theory can be applied to any simplicial and regular CW-complexes and, although its definition is quite simple, many classical results analogous to the ones of the continuous Morse theory arise in its scope: it allows to compute homologies, cup-product, Novikov homologies, develop Witten's deformation of the Laplacian, etc..

The classical construction of barycentric subdivision of simplicial complexes can be used to approximate a smooth structure on a triangulated topological manifold. We develop a simple algorithm to "transfer" a discrete Morse function, defined on a simplicial complex, onto the barycentric subdivision of this complex in such a way that all important data about this function (i. e. the number and dimensions of the critical simplexes and the gradient path structure) stays unchanged [2]. It can be done in several different ways (they differ inside the critical simplexes of the initial field) and so we can produce several different Morse functions.

References

- [1] R. Forman, *Morse Theory for Cell Complexes*. Advances in Mathematics, 134, 90–145 1998.
- [2] A. Zhukova, *Discrete Morse theory for the barycentric subdivision*, preprint (2016), available at <https://arxiv.org/abs/1605.04751>.

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