Further Algebraization of Differential Equations and Applications

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Abstract. We propose a method of calculation of differential equations for coefficients of the complicated expansions of solutions to a differential equation. The method uses the higher variations. It was applied to the Painlevé equations.

1. General situation

The Taylor series gives expansions of values of a polynomial using higher derivatives. I propose to use similar expansions of values of a differential polynomial using higher variations, i.e. the Freche/Gato derivatives.

2. Ordinary differential equations

Some ordinary differential equations have asymptotic expansions of solutions as a power series which coefficients are the Laurent series of logarithms (so called complicated expansions or psi-series). The mentioned approach allows to calculate differential equations for each such coefficient separately using higher variations of parts of the initial equation. These equations for coefficients can be solved by the known methods. But sometimes the coefficients are polynomials instead of to be Laurent series.

3. Painlevé equations

Among six Painlevé equations three of them have such complicated expansions of solutions: P_3, P_5 and P_6. Their first coefficients are polynomials of logarithms. In examples 3 and 4 of [1] there were computed some second coefficients for P_3 and P_6. They are polynomials. Recently I computed all second coefficients for P_3 and P_6. All of them are polynomials. Now I computed the third coefficients for P_3 and

P_6. They are polynomials only under some restrictions on four parameters of the equations in form of polynomial equations. Sometimes the third coefficients are not polynomial at all.

References

[1] A. D. Bruno, *On complicated expansions of solutions to ODE,* Preprint of KIAM No. 15. Moscow, 2011. 26 p. (Russian)

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