Diagonal complexes

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Assume that $n > 2$ is fixed. We say that two diagonals in a convex n-gon are *non-intersecting* if they intersect only at their endpoints (or do not intersect at all). John Milnor showed that the poset of all collections of pairwise non-intersecting diagonals in the n-gon (ordered by reverse inclusion) is isomorphic to the face poset of some convex $(n-3)$-dimensional polytope As_n called *associahedron*.

Instead of a polygon let us take an arbitrary (possibly bordered) orientable surface with a number of marked points (=vertices) lying not necessarily on the boundary. Generalizing a construction of J.L. Harer, we introduce and study similar diagonal complexes $\mathcal{C}$ and $\mathcal{B}$. Investigation of some natural forgetful maps combined with length assignment proves homotopy equivalence for some of the complexes, for the space of metric ribbon graphs $RG_{g,n}^{met}$, for the tautological S^1-bundles L_i, and for a more sophisticated bundle whose fibers are homeomorphic to some surgery of the surface F. The latter is shown to incorporate all the tautological S^1-bundles.

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