# Equal Mass Free-Fall Three-Body Problem: Symbolic Dynamics, Numerical Inverstigation

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**Abstract.** We consider equal mass free-fall three-body problem. We construct numerically symbolic sequences using close binary approaches and analyze two components revealed as peaks on the histogram for the Shannon entropy of these sequences.

## Introduction

Symbolic dynamics was used to analyze some special cases of the three-body problem: Alexeyev [2, 3, 4, 5] has found an intermittence of motions of different types in the one special case of the three-body problem - Sitnikov problem. Symbolic dynamics was also applied in two other special cases of the three-body problem: the rectilinear problem (Tanikawa & Mikkola [10, 11]); and the isosceles problem (Zare & Chesley [13, 6]). Tanikawa & Mikkola [12] considered the case with non-zero angular momentum.

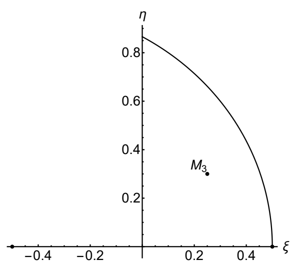


FIGURE 1. Agekian-Anosova region D.

Equal mass free-fall three-body problem is convenient for study since it allows easy visualization of initial configuration: if we place two bodies in the points

$(-0.5;0)$ and $(0.5;0)$, then all possible configurations will be covered if we place the third body inside the region D bounded by two straight line segments and arc of the unit circle centered at $(-0.5,0)$ (Fig. 1) [1]. Here, we analyze components revealed as two peaks on the histogram for the Shannon entropy of the symbolic sequences constructed using close binary approaches that we found earlier [9]. Raspberry Pi cluster was used for numerical integration of trajectories and construction of symbolic sequences, Wolfram Mathematica is used to analyze sequences received. We used symplectic code by Seppo Mikkola (Tuorla Observatory, University of Turku) [8] for numerical simulations.

## 1. Construction of symbolic sequences

We cannot establish homomorphism between individual trajectories of the three-body system and symbolic sequences in the general case, so we have to integrate equations of motion numerically and construct symbolic sequences during the process.

Final stage of the evolution of typical three-body system is close binary while the third body is ejected from the system. So, all symbolic sequences have predictable final parts. If one will calculate entropy of such "infinite" (long enough in practice) sequence, the result is also obvious. So, we study the evolution of the system during the finite period of time, considering the stage of active interaction between the bodies. Thus, we study complexity of finite sequences. One can say that in our "numerical symbolic dynamics" approach we replace original three-body system by a dynamical system that behaves like our original system during this period of time, and have similar behavior all other time (without disruption).

One can use different methods to construct symbolic sequences (see e.g. [9]). In this study we construct symbolic sequences using binary encounters: we detect minimum distance between two bodies, and corresponding symbol is the number of the distant body. Thus, our symbols are from the alphabet $\{1, 2, 3\}$. Some systems disrupt fast, so some sequences are short. Some systems live long (e.g. metastable systems [7]), so corresponding sequences are long. To have a reasonable computing time, we constructed symbolic sequences length 50. Since we are interested in the analysis of active three-body interactions, we consider sub-sequences of each of these sequences, increasing the length step-by-step, calculate entropy for each of these sub-sequences, and find maximum value of these entropies. Maximum value (and moment of time/length of the sub-sequence) correspond to the stage of active interaction between bodies.

Histogram of maximum values of the entropy shows two distinct modes (Fig. 2). Interesting structures can also be seen on the scatterplot of maximum values of the entropy - corresponding length of symbolic sequence in the neighborhood of these modes (Fig. 3). Left mode corresponds to the sequences with only two

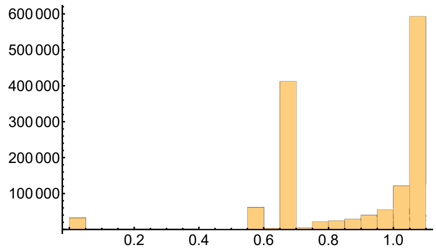


FIGURE 2. Histogram of maximum values of the entropy.

symbols equally represented: Entropy[{1, 2, 1, 2}]=0.693147. Second mode corresponds to the sequences where all three symbols are equally presented: Entropy[{1, 2, 3, 1, 2, 3}]=1.09861.

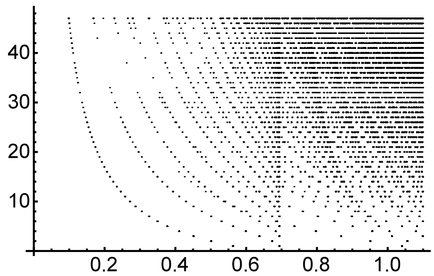


FIGURE 3. Scatterplot of maximum values of the entropy - corresponding length of symbolic sequence.

To reveal the difference between sequences of the type {1,1,1,2,2,2,3,3,3} and {1,2,1,3,2,3,2,1,3} one can use Markov entropy $H = -\sum_i p_i \sum_j q_{ij} \ln q_{ij}$. Here $p_i$ is the frequency of symbol "$i$" in the sequence, and $q_{ij}$ is the frequency of transitions from "$i$" to "$j$". Another option is to form pairs (triples, etc.) of consecutive symbols and calculate corresponding entropies. Histogram of the entropy for sequences constructed from the pairs of symbols for systems from the second invariant component (right mode on Fig. 2) reveals that there is large enough number of systems with good triple interactions (Fig. 4).

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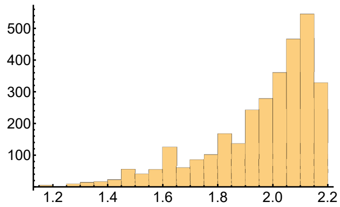


FIGURE 4. Histogram of values of the entropy for pairs of symbols for sequences from the neighborhood of the right mode on Fig. 2.

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