Zero divisors in group rings and combinatorial corollaries

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Let G be a finite group, K a field and $X_1, \ldots, X_k$ be subspaces of the group algebra $K[G]$ such that $X_1 \ldots X_k = 0$. If the codimensions of X_i are small, it implies many strong combinatorial properties of G, like the following:

any product set $A_1 \ldots A_{k-1}$ may be covered by a set C of size $\operatorname{codim} X_k$ and $k-1$ smaller product subsets $A_1 \ldots A_{i-1} B_i A_{i+1} \ldots A_{k-1}$ for certain $B_i \subset A_i$, $|B_i| \leqslant \operatorname{codim} X_i$. This generalizes the previous results by Ellenberg and Kleinberg, Sawin, Speyer, which all originated in a breakthrough paper by Croot, Lev and Pach.

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