Applications of Steiner symmetrization to some extremal problems in geometric function theory

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We investigate properties of the Steiner symmetrization in the complex plane. We use two recursive dynamic processes in order to derive some sharp inequalities on analytic functions in the unit disk. We answer a question that was asked by Albert Baernstein II, regarding the coefficients of circular symmetrization. We mostly deal with the Steiner symmetrization G of an analytic function f in the unit disk U. We pose few problems we can not solve. An intriguing one is that of the inequality

$$\int_0^{2\pi} |f(re^{i\theta})|^p d\theta \leq \int_0^{2\pi} |G(re^{i\theta})|^p d\theta, \ 0 < p < \infty$$

which is true for $p = 2$ (we prove) but can not be true for too large p. What is the largest such exponent or its supremum?

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